Landauer principle and reversible logic

M. Cristina Diamantini

Nips laboratory, INFN and Department of Physics and Geology

University of Perugia

Landauer's principle, often regarded as the basic principle of the thermodynamics of information processing, holds that any logically irreversible manipulation of data ...must be accompanied by a corresponding entropy increase in non-information bearing degrees of freedom of the information processing apparatus or its environment.
Conversely, it is generally accepted, any logically reversible transformation of information can in principle be accomplished by an appropriate physical mechanism operating in a thermodynamically reversible fashion
Bennet,2003

….. logically irreversible operations can be implemented without generating heat.
Any logically irreversible transformation of information can in principle be accomplished by an appropriate physical mechanism operating in a thermodynamically reversible fashion"
"………it is possible to show that information and entropy, while having the same form, are conceptually different
Maroney,2008

Possibilities	Thermodynamically reversible	Thermodynamically irreversible
Logically reversible		
Logically irreversible		

Can we perform logically irreversible computation in a thermodynamically reversible manner?

Can we perform logically irreversible computation without dissipating heat in the environment?

Are these two questions two different questions?

O.J.E. Maroney: Studies in History and Philosophy of Modern Physics vol. 36 pp. 355-374 (2005); Phys. Rev.E 79, 031105 (2009)
T. Sagawa: J. Stat. Mech. P03025 (2014)

- Landauer principle: information is physical
  →information theory
  →thermodynamics

- Relevant for the engineering of devices used to process information:
  Landauer principle: fundamental bound on energy dissipation ⇒ energy consumption:
  Landauer bound + how good we are in engineering devices

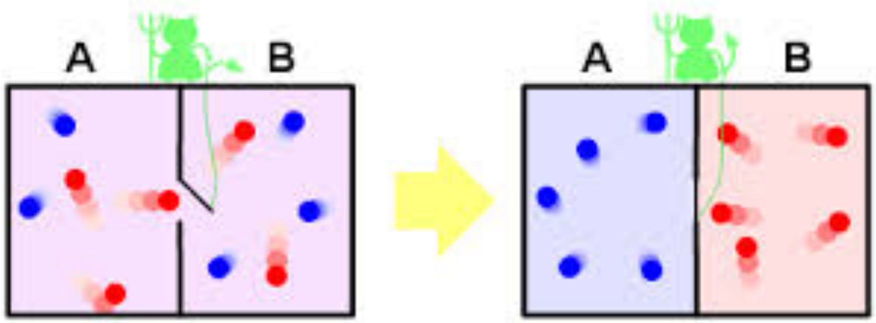
  ~~Landauer~~ bound + how good we are in engineering devices?

  (at least in principle)

- Physical systems obey the II law of thermodynamics, always true?
  →Maxwell demon
  →Szilard engine

Maxwell demon

Gas in a partitioned box (Maxwell 1861)



demon can measures the speed of the molecules (thermodynamically reversible)

Information is gained

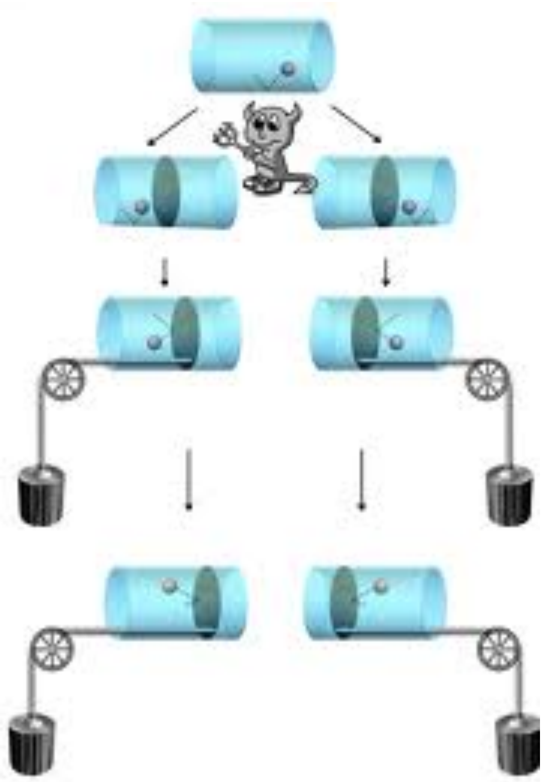
→sorting of the molecules

→temperature difference (decrease of entropy without work) that can be used to produce work

→apparent violation of the second law

Szilard engine

- One-particle gas in a partitioned box (Szilard 1929)



Initial probability ½ of the atom being on either side of the barrier

The demon performs a measure to determine the position of the particle (no heat production): information is gained

The piston is moved to one side or the other: extract work $W = kT \ln 2$ that can be used to lift a weight

Apparent violation of the II law

Landauer principle

Resolution of the puzzle: forgetting is costly !

(Landauer 1961)

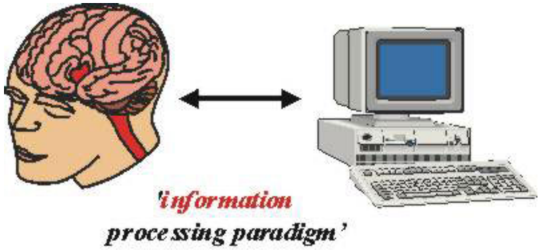
- Szilard engine: the Demon has a single memory register (0,1), initially 0; after measure it represents the location of the particle, 0 or 1

- To complete the cycle the memory of the Demon must be erased (information lost)

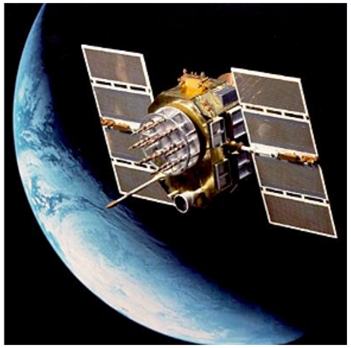
- Landauer principle: erasure of information requires a minimun heat production: $\langle Q \rangle \geq \langle Q_{Landauer} >= kT \ \ln 2$ per bit $\Rightarrow$
  the conversion of kTln2 work into heat compensats the work extracted!

  - Information is physical

- information is processed trough physical systems



- Information is stored in physical systems



- Information is transmitted trough physical systems

- Experimental verification:
  - ✓ Szilard engine: brownian particle in a tilted periodic potential (Toyabe et al. Nature 2010) →demonstration of information to work conversion
  - ✓ Landauer principle: brownian particle in a double-well potential (Berut et al. Nature 2012)
    →mean dissipated heat saturates Landauer bound

What is Information?
(Shannon 1948)

Common language: spin can be up or down ⇒ the only instruction we need to transmit to recreate the state is whether the state is spin-up or spin-down

- Information content of an object = the size of the set of instructions required to reconstruct the state of the object
- a complicated set of instructions can be reduced to n binary choices → information content = number of binary choices i.e. number of bits ≡ classical variables that can assume only the values 0 or 1 with probability $p(0) = p_0$ and $p(1) = p_1$ (alphabet with 2 letters), $\sum_I p_i = 1$
- Construct a measure for the amount of information associated with a message: the less likely a message is, the more info gained upon its reception!

The amount of info gained from the reception of a message depends on how *likely* it is (Shannon)

Shannon Entropy

- Measure the "surprise" content of a letter i with probability p_i: $\log 1/p_i \geq 0$

  1. additive: $\log (1/p_i\ 1/p_{j)} = \log 1/p_i + \log 1/p_j$
  2. the greater p_i is, the more certain the letter i $\Rightarrow$ less information should be associated with it.

- Compute the average "surprise" :
  $H = \sum^1_{i=0} p_i \log 1/p_i = - \sum^1_{i=0} p_i \log p_i \geq 0$ Shannon entropy

- Generalize to an alphabet of M letters, with probability p(m), $m \in M$, and $\sum_{m\in M} p(m) = 1$ and $p(m) \geq 0$:
  $H(M) = - \sum_{xm\in M} p(m) \log p(m)$

- H(M) tells us our expected information gain upon measuring M

(In the following $\log_2 \rightarrow \ln$ and drop the ln2)

Conditional entropy and Mutual Information

- joint probability p(m,n) of two events m, n is the probability of event m occurring at the same time event n occurs;
  $p(n) = \sum_{m \in M} p(m,n)$; $p(m) = \sum_{n \in M'} p(m,n)$
  $p(m,n) = p(m)p(n)$, m and n independent
  → $H = -\sum_{m \in M} \sum_{n \in M'} p(m,n) \ln p(m,n)$ joint entropy
- conditional probability p(n|m): probability of the event n, given that event m has occurred; $p(m,n) = p(n|m)\, p(m)$
  → $H(M'|M) = -\sum_{m \in M} p(m) \sum_{n \in M'} p(n|m) \ln p(n|m)$ conditional entropy
  H(M'|M) measures the average uncertainty about the value of an output, given a particular input value.

Example: Drawing 2 Kings from a Deck, m= drawing a King first, n =drawing a King second
$p(m) = 4/52$ (there are 4 Kings in a Deck of 52 cards)
after removing a King from the deck the probability of the 2nd card drawn is less likely to be a King (only 3 of the 51 cards left are Kings) $\Rightarrow p(n|m) = 3/51$
$p(m,n) = p(n|m)\, p(m) = (4/52) \times (3/51) = 1/221 \approx 0.5\%$

- Bayes' theorem: $p(m|n) = (p(n|m)\, p(m)) / p(n)$

- relative entropy or Kullback-Leibler distance between two distribution p(m) and q(m): $D(p||q) = \sum_{m \in M'} p(m) \ln (p(m)/q(m)) \geq 0$

- Mutual information I(M;M') = relative entropy between joint distribution and product distribution:
  $I(M;M') = \sum_{m \in M} \sum_{n \in M'} p(m,n) \ln p(m,n) / (p(m)\, p(n))$
  is a measure of the amount of information a random variable contains about another random variables
  m, n independent $\Rightarrow p(m,n) = p(m)\, p(n)$
  $\Rightarrow I(M;M') = 0$

- $I(M;M') = H(M) - H(M|M') = H(M') - H(M'|M) = H(M) + H(M') - H(M,M')$
  $I(M;M') = I(M';M)$; $I(M;M) = H(M)$

- If m is a continuous variable the sums are replaced by integrals

Logical Operations

- A logical operation maps a set of inputs states $m \in M$ to a set of output states $n \in M'$: LO: $m \rightarrow n$

- Logically reversible operations: the output of the devices uniquely defines the input:
  $\forall$ m, n $p(m|n) \in \{0,1\}$ or $\forall$ n : $p(n|m) \neq 0 \Rightarrow p(n|m')=0 \ \forall \ m' \neq m$

- Logically deterministic operations: the input of the devices uniquely defines the output:
  $\forall$ m, n $p(n|m) \in \{0,1\}$ or $\forall$ m : $p(m|n) \neq 0 \Rightarrow p(m|n')= 0 \ \forall \ n' \neq n$

1. logically reversible deterministic: NOT: 0 (1) → 1(0)
   $\Delta H = 0$
2. logically irreversible deterministic: RTZ: {0,1} → 0
   $\Delta H < 0$

3. logically reversible indeterministic: UFZ: 0 → {0,1}
   $\Delta H > 0$
4. logically irreversible indeterministic: RND: {0,1} → {0,1}
   $\Delta H = 0$ (if $p(0)= p(1) = ½$ both for input and output states)

- Why add them:

1. significant role in the theory of computational complexity classes for actual computers; in some cases inclusion of logically indeterministic operations can produce an accurate answer exponentially faster than any known algorithm consisting only of logically deterministic operations

2. more coherent general framework for the thermodynamics of computation; properties ascribed to logically irreversible operations may be artefacts of the asymmetry caused by exclusion;

3. no special reason not to include, natural counterpart to the concept of logically irreversible operations; conclusions that applies to the set of all such logical operations must necessarily apply to all logically deterministic operations

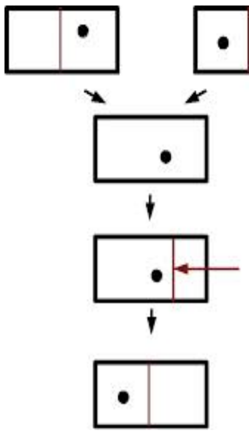
One-bit erasure (RTZ)

Two logical states memory: {0,1} → 0

- Initial configuration: two logical states with equal probability ½
  Shannon entropy $H_i = -\sum_I p_i \ln p_i = \ln 2$
- Final configuration: one logical state with probability 1, $H_f = 0$
- $\Delta H = -\ln 2$
- More logical input states then output → the computational process is not an injection → logically irreversible
  Information is physical → II law of thermodynamic
  $\Delta S_{tot} = \Delta S_{syst} + \Delta S_{res} \geq 0$; $T \Delta S_{res} = Q_{res}$ $\Delta S_{syst} = -k T \ln 2$
- Heat is produced during a logically irreversible computation

$$Q_{res} \geq k T \ln 2 = - k T \Delta H$$

Thermodynamic reversibility



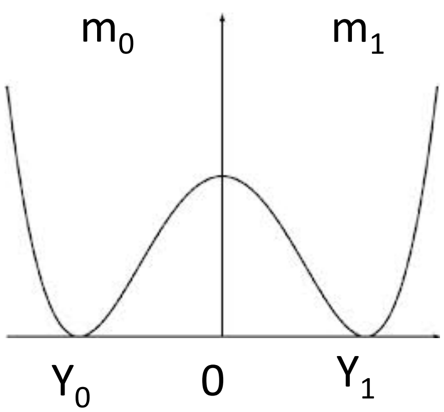
- Work done during a quasi-static isothermal erasure ideal gas
  step1: free expansion W= 0
  step 2: isothermal quasi-static compression: $W= -\int_V^{V/2} PdV = -NkT[\ln(V/2) - \ln V] =$
  $= NkT \ln 2$; $\Delta U = 0 \Rightarrow Q_{sys} = -W$
- A process is thermodynamically reversible if $\Delta S_{tot} = \Delta S_{syst} + Q_{res}/T = 0$
- in this case $\Delta S_{tot} = k(-\ln 2) + k(\ln 2) = 0$ thermodynamically reversible
- $Q_{res} = kT \ln 2$ during a quasi-static computational process, no contradiction with Landauer principle

⚠ But this does not mean that you do not generate heat!

- At finite velocity $Q_{res} > kT \ln 2$
- Thermodynamic and logical reversibility are not equivalent!

Physical implementation of the memory

- Y phase space of the memory ⇒ $y \in Y$ microscopic physical state of the memory; M set of possible logical states, $m \in M$ logic state; in general we have many physical states that correspond to a single logical state



- Two states memory implemented through a symmetric bistable potential: left well ($y< 0$) logical state m_0; right well ($y\geq 0$) logical state m_1

- Physical phase space: $Y = \cup_{m \in M} Y_m$
- $Y_m \cap Y_n = \varnothing \; m \neq n$ to ensure that one phase space point belongs to only one logical state ⇒ $p(y|m) \neq 0 \leftrightarrow y \in Y_m$

Input state:

- $m \in M$ logical state with probability $p(m)$,
- $y \in Y_m$ initial phase-space point with probability $p(y)$,
- $p(m) = \int_{y \in Y} dy\ \ p(y|m)p(y) = \int_{y \in Y_m} dy\ p(y)$ since $p(y|m) \neq 0 \leftrightarrow y \in Y_m$
- $p(y) = \sum_{m \in M} p(y|m)\ p(m)$

Output state:

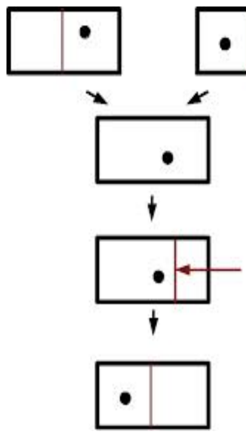
- $n \in M'$ logical state with probability $p(n)$,
- $y' \in Y_n$ final phase-space point with probability $p(y')$,
- $p(n) = \int_{y' \in Y} dy'\ \ p(y'|n)p(y) = \int_{y' \in Y_n} dy'\ p(y')$ since $p(y'|n) \neq 0 \leftrightarrow y' \in Y_n$
- $p(y') = \sum_{n \in M'} p(y'|n)\ p(n)$

Logical operation LO: $m \rightarrow n$ implemented through a physical transformation from the initial state $y \in Y_m \rightarrow y' \in Y_n$

reliability of the implementation consists …(in ensuring) that whichever of the representative physical states (physical state of the device that corresponds to the possible logical states) the device is prepared in, it ends up in the appropriate representative state (Ladyman et al.)

a physical process is a change in a physical system whereby it goes from a particular physical state to a particular physical state. Hence, strictly speaking a physical process cannot be said to implement a logical transformation because all it could ever do is implement the part of the map that takes one of the logical input states to another logical input state (Ladyman)

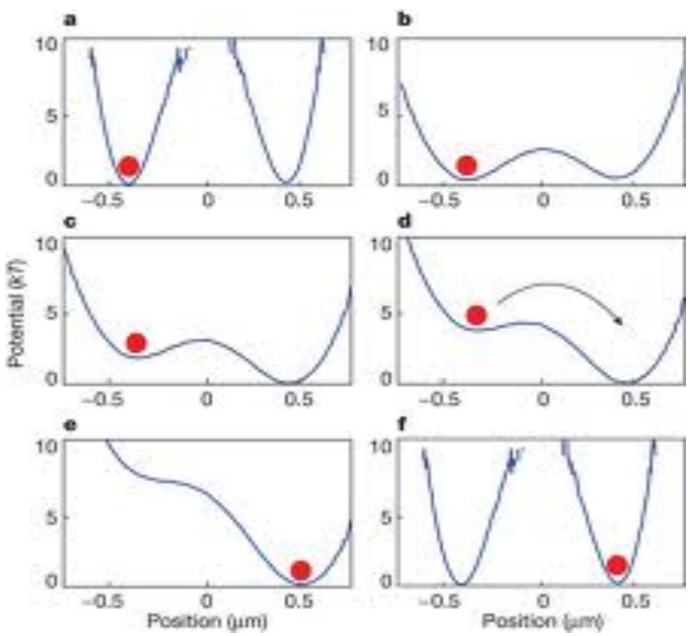
- Particle in a box:



Removal of the barrier
(memory is erased, but not reset)

(if we know the position, Feynman Lectures on Computation)

- Symmetric bistable potential



Removal of the barrier
(memory is erased, but not reset)

Berut et al.

Entropy

- Physical system implementing the memory in contact with a heat bath at temperature T $\Rightarrow S_{tot} = S_{sys} + S_{res}$

- Entropy of the physical system, Gibbs entropy:
  $S_{syst}(Y) = - k \int_{y \in Y} dy \; p(y) \ln p(y)$

- Logical entropy : $H(M) = - \sum_{m \in M} p(m) \ln p(m)$

- These two entropies are related:
  $S_{syst}(Y) = k H(M) + S(Y|M)$
  $S(Y|M) = \sum_{m \in M} p(m) S(Y|m)$; $S(Y|m) = -k \int_{y \in Ym} dy \; p(y|m) \ln p(y|m)$

- In fact:

$S_{syst}(Y) = -k \int_{y \in Y} dy\ p(y) \ln p(y) =$

$= -k \int_{y \in Y} dy \sum_{m \in M} p(y|m)\, p(m) \ln \sum_{n \in M} p(y|n)\, p(n)$

but $p(y|m) \neq 0 \leftrightarrow y \in Y_m \Rightarrow$

$\Rightarrow - \sum_{m \in M} k \int_{y \in Ym} dy\ p(y|m)\, p(m) \ln p(y|m)\, p(m) =$

$= - \sum_{m \in M} k \int_{y \in Ym} dy\ p(y|m)\, p(m) \ln p(y|m) +$

$- \sum_{m \in M} k \int_{y \in Ym} dy\ p(y|m)\, p(m) \ln p(m) =$

(since $\int_{y \in Ym} dy\ p(y|m) = 1$)

$= -\sum_{m \in M} p(m)\, S(Y|m) \quad -k\, H(M) =$

$= k\, H(M) + S\,(Y|M)$

- fluctuations over the whole phase space (physical device) = fluctuations of logical states + fluctuations in the physical space corresponding to individual logical subspace

Generalized Landauer principle

- Entropy variation during computation (Sagawa, Maroney):

  $\Delta S_{syst} = k\,\Delta H + \Delta S_{cond}$

  with $\Delta S_{syst} = S'_{syst}(Y) - S_{syst}(Y)$;

  $\Delta H = H'(M') - H(M)$; $\Delta S_{cond} = S'(Y|M') - S(Y|M)$

- II law of thermodynamic: $\Delta S_{tot} = \Delta S_{syst} + \Delta S_{res} \geq 0 \Rightarrow$

  $\Rightarrow \Delta S_{tot} = k\,\Delta H + \Delta S_{cond} + Q_{res}/T \geq 0$

- $Q_{res} \geq - k\,T\,\Delta H - T\,\Delta S_{cond}$ generalized Landauer limit

  the change in the Shannon entropy during a logically irreversible computation can be compensated by the increase in the entropy of the heat bath and by the increase in the entropy of physical states inside a logical subspace

- $\Delta S_{cond} + Q_{res}/T \geq -k\,\Delta H$ a transformation of information requires an increase of entropy of the non-information bearing degrees of freedom (NIBDF) of at least the change in the Shannon entropy

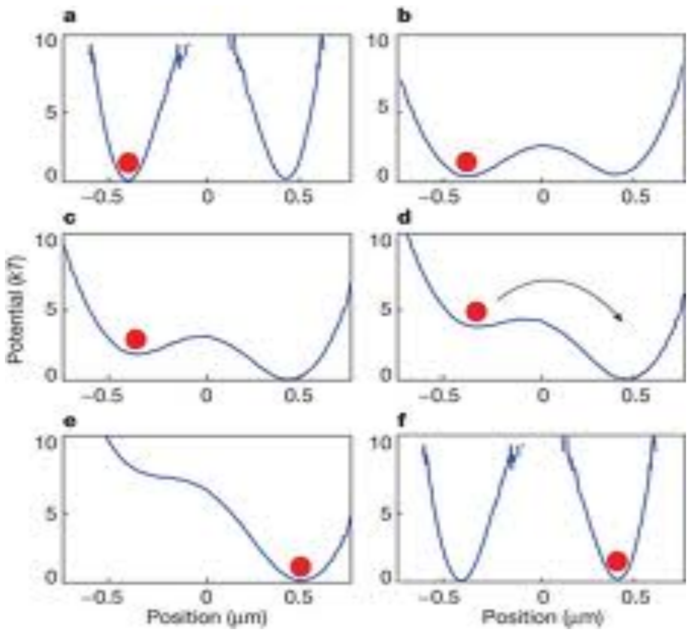
- possibility of realizing erasure process with less heat emission:
  $Q_{res} \geq -k\,T\,\Delta H - T\Delta S_{cond}$
  $Q_{res} \geq 0$ if $-k\,\Delta H = \Delta S_{cond}$

- if $Q_{res} = -kT\Delta H - T\,\Delta S_{cond} \Rightarrow \Delta S_{tot} = 0$, lower bound satisfied in $\Delta S_{tot} \geq 0 \Rightarrow$ the computation is thermodynamically reversible (quasi-static limit)

- thermodynamic reversibility $\equiv$ lower bound satisfied in $\Delta S_{tot} \geq 0$

- $Q_{res} \geq - k\, T\Delta H$ if $\Delta S_{cond} = 0$ conventional Landauer limit

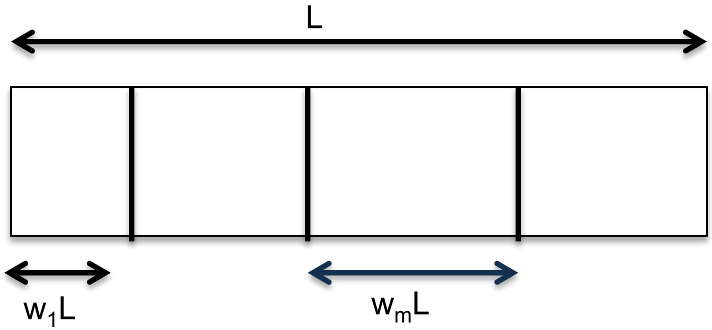
- Why in the theoretical and experimental verifications of Landauer principle $\Delta S_{cond} = 0$?
  general setup for one bit erasure
  (e.g.: Berut et al.; Piechocinska)



- symmetric bistable potential with a barrier of height ΔU
  $\Delta U >> kT \Rightarrow \Delta S_{cond} = 0$
  (more in the talk of Dr Chiuchiu')

- optimization of the thermodynamic implementation

Optimization (Maroney)



- Generalized particle in a box model:
  transition m → n: p(n|m)
  p(m,n) = p(n|m) p(m)

- variation of w_m used to optimize the process subject to the constraints: $\sum_{m \in M} w_m = 1$ and $w_n = \sum_{m' \in M} w_{m'}\, p(n|m')$

- minimum heat generated (average over input probability distribution) for
  $p(m) = w_m$:
  $\Delta Q = -T\,(\Delta S_{cond} + k\,\Delta H)$ optimum process
  $\Delta Q > -T\,(\Delta S_{cond} + k\,\Delta H)$ non-optimum process

- true for logical reversible, irreversible, deterministic and indeterministic logical operations

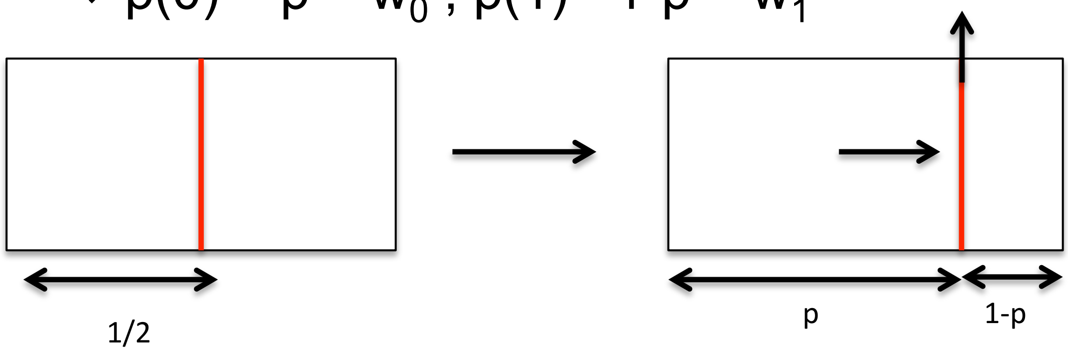
- Optimum process involve various idealizations, e.g. frictionless motion and quasi-static processes ⇒ not achievable in practice, but possible in principle

- Thermodynamically optimization of physical process⇒ knowledge of probability distribution P(m) over all logical input states

- Optimum physical implementation for one input probability distribution ⇏ optimum for different input probability distribution

- No physical process can implement the same logical transformation with a lower expectation value for work requirement or heat generation

- Example:
  Landauer erasure with non-uniform probabilities (particle in a box)
- input: 0 with probability p ,1 with probability $(1-p)$
  ouput: 0 with probability 1
- $\Delta H = [\, p \ln p + (1-p) \ln (1-p)]$
- add step (step 0) to Landauer erasure (before removing the barrier)
  $\Rightarrow p(0) = p = w_0$, $p(1) = 1-p = w_1$



$W_p = -kT[p \ln p + (1-p) \ln (1-p) + \ln 2]$

$W_{tot} = W_p + kT \ln 2 = kT\Delta H \Rightarrow \Delta S_{tot} = 0$ reversible

- Apply the procedure optimize for probabilities $p(0) = p(1) = 1/2$
  (no step 0) $\Rightarrow W_{tot} = kT \ln 2$
- $\Delta S_{tot} = k\ \Delta H + k \ln 2 = k[\, p \ln p + (1-p) \ln (1-p)] + k \ln 2 > 0$ for $p \neq 1/2$
  not reversible, $\Delta S_{tot} = 0$ if $p = 1/2$
- Removal of the partition at $x=1/2$ when probability is p is associated with an uncompensated entropy increase

Thermodynamic versus Logic Reversibility

It is always possible during a computational process to satisfy the lower bound in the inequality $\Delta S_{tot} \geq 0$? The answer depends on what is $\Delta S_{tot} \equiv$ what is the thermodynamic entropy

We considered $\Delta S_{tot} = \Delta S_{syst} + \Delta S_{res}$ with $\Delta S_{syst} = \Delta S_{gibbs}$

$\Delta S_{gibbs} = \Delta S_{cond} + k\Delta H$

In this case it is always possible to have $\Delta S_{tot} = 0$

Reversibility

↙ ↘

Logic : determined by change of the entropy in the logical states

Thermodynamic: determined by change of the entropy of the universe, physical system + heat bath (ensemble)

Alternative definition (Maroney)

- Thermodynamic entropy is the entropy of individual state: S(Y|m) if the system is in the m logical state; for a transition m→n, that occurs with probability p(m|n),

  $\Delta S_{tot(m,n)} = S'(Y|n) - S(Y|m) + Q_{m,n}/T$ (NIBDF)
- On average $\Delta S_{tot} = \Delta S_{cond} + Q/T \equiv \Delta S_{NIBDF}$
- II law: $\Delta S_{NIBDF} \geq 0$
- $\Delta S_{NIBDF} = -k\,\Delta H$ (optimization of the thermodynamic cost of the transition)
- Reversible deterministic logical operations: $\Delta H = 0$

  (injection between input and output states) $\Rightarrow$

  $\Rightarrow \Delta S_{NIBDF} = 0$ thermodynamically reversible
- irreversible deterministic logical operations: $\Delta H = < 0$ (more input states corresponds to the same output state) $\Rightarrow$

  $\Rightarrow \Delta S_{NIBDF} > 0$ thermodynamically irreversible

What happens if we consider indeterministic operations?

- UFZ: 0 → {0,1} , if $p(0) = p(1) = ½$ in the output $\Rightarrow \Delta H > 0 \Rightarrow \Delta S_{tot} \equiv \Delta S_{NIBDF} < 0$ wrong measure of the entropy !

- Gibbs measure of the entropy, that takes into account the effect of the statistical mixture over the states gives $\Delta S_{tot} \geq 0$

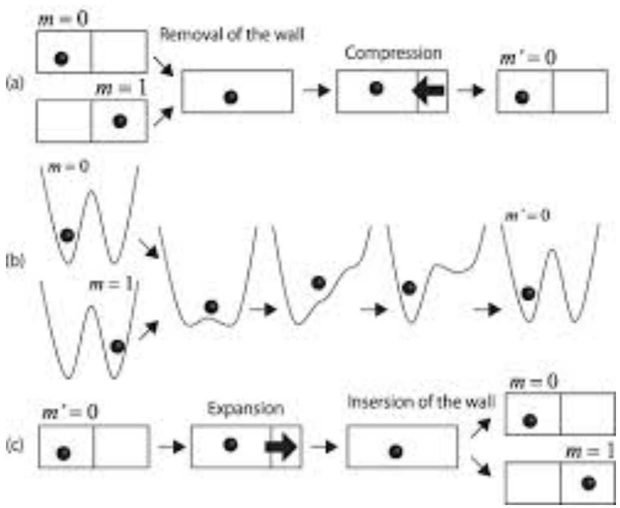
- Indeterministic operations should not be taken into account
  logical operation := single valued map
  (Ladyman, Presnell, Short, Groisman)

……more concrete statement of the second law of thermodynamics which is usually referred to as the Kelvin formulation:
"It is impossible to perform a cyclic process with no other result than that heat is absorbed from a reservoir, and work is performed." (Uffink 2001, p. 328)

- $\Delta Q_{res} /T = = \Delta S_{NIBDF} + k \Delta H \geq 0 \Rightarrow \Delta S_{NIBDF} \geq -k \Delta H > 0$ since the logical operation is irreversible ⇒ thermodynamic irreversibility

LE + RLE Cycle

RTZ + UFZ

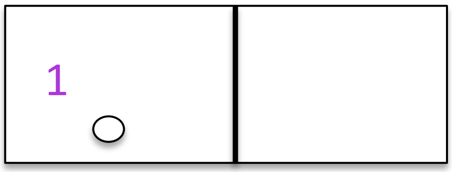


- RTZ (LE): initial probability uniformly distributed (p(0) = p(1) =1/2)
  during the compression work is done on the system, work required kTln2
- UFZ (RLE): during the expansions work is extracted from the system, work extracted kTln2
  final probability uniformly distributed (p(0) = p(1) =1/2)

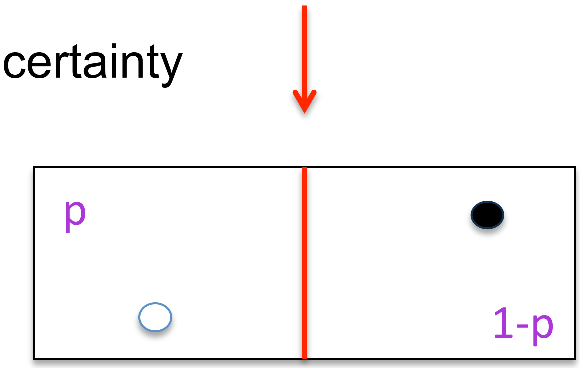
- The work done to reset the bit during LE is recovered during the expansion in the RLE ⇒ the entire process is thermodynamically reversible
- If initial distribution in the RTZ and final distribution in the UFZ match ⇒ LE + RLE is a reversible cycle

memory erasure and resetting is always logically irreversible but it is thermodynamically reversible only when the initial memory ensemble is distributed uniformly among 0 and 1 states (Leff and Rex)

LE+RLE Cycle
Non-uniform Probabilities



- Particle on the left side with certainty



- RLE(p):
- Input: 0 with probability p(0) =1
  Output: 0 with probability p(0) =p ; 1 with probability p(1) =1 - p

1. Isothermally move the partition to the right W_1= -kT ln2 (extract work)
2. Insert the partition at x=p
3. Isothermally move the partition to the center,
   mean work W_2= kT[p ln p + (1- p) ln (1- p) +ln 2]
   $W_1 + W_2 =$ kT[p ln p + (1- p) ln (1- p)] ≤ 0

- LE(p):

1. Isothermally move the partition to the position x=p, mean work $W_3 = -kT[p \ln p + (1-p) \ln (1-p) + \ln 2]$
2. Remove the partition
3. Insert the partition to the right and isothermally move it to the center , $W_4 = kT\ln 2$

- $W_3 + W_4 = -kT[p \ln p + (1-p) \ln (1-p)]$

- $W_1 + W_2 + W_3 + W_4 = 0$

- RLE and LE optimized for the same probability distribution ⇒ reversible cycle

- RLE and LE optimized for different probability distributions:
  RLE(p) + LE(p')

- LE(p'):

1. Isothermally move the partition to the position x=p', mean work
   $W_5 = -kT[p \ln p' + (1- p) \ln (1- p') + \ln 2]$
2. Remove the partition
3. Insert the partition to the right and isothermally move it to the center,
   $W_6 = kT\ln 2$
   $W_5 + W_6 = - kT[p \ln p' + (1- p) \ln (1- p')]$

- $W_1 + W_2 + W_5 + W_6 = kT [p \ln (p/p') + (1- p) \ln ((1- p)/(1-p'))] \geq 0$
- Equality occurs when p= p'
- The cycle is thermodynamically irreversible for $p \neq p'$
- Removal of the partition at x=p' when probability is p is associated with an uncompensated entropy increase

Adiabatic computing

- Optimum process (Maroney):
  $\Delta Q_{res} \geq - T\ (\Delta S_{cond} + k\ \Delta H\ ) =$
  $= -\ T[\ \sum_m p(m)[\ (S(Y|m)\text{-}k\ \ln\ p(m)] - \sum_n p(n)[\ (S'(Y|n)\text{-}k\ \ln\ p(n)]\ ]$

  $\Delta W \geq \sum_m p(m)[\ E_m\text{-}\ T\ (S(Y|m)\text{-}k\ \ln\ p(m)] - \sum_n p(n)[\ E_n\text{-}\ T\ (S'(Y|n)\text{-}k\ \ln\ p(n)]$

- Uniform computing: isothermal and physical states that represents logical states have same entropy and mean energy
  $\sum_n p(n)\ E_n = \sum_m p(m)\ E_m \qquad \sum_m p(m)\ S(Y|m) = \sum_n p(n)\ S'(Y|n) \Rightarrow$
  $\Delta W \geq -\ kT\ \Delta H$

- If $\sum_m p(m)[\ E_m\text{-}\ T\ (S(Y|m)\text{-}k\ \ln\ p(m)] = \ \sum_n p(n)[\ E_n\text{-}\ T\ (S'(Y|n)\text{-}k\ \ln\ p(n)]$
  $\Rightarrow\ \Delta W \geq 0 \quad \Delta Q_{res} \geq T\ \Delta S_{sys}$
  if $\sum_m p(m)[\ (S(Y|m)\text{-}k\ \ln\ p(m)] = \ \sum_n p(n)[\ (S'(Y|n)\text{-}k\ \ln\ p(n)]$
  $\Rightarrow \Delta Q_{res} \geq\ 0 \quad \Delta W \geq \Delta U$
  (E_m- T (S(Y|m)-k ln p(m) = C a device dependent constant, input and output states canonically distributed)

- Adiabatic equilibrium computing:

1. $\sum_m p(m)[\, E_m - T\,(S(Y|m) - k \ln p(m))] = \sum_n p(n)[\, E_n - T\,(S'(Y|n) - k \ln p(n))]$
2. $\sum_m p(m)[\, (S(Y|m) - k \ln p(m)] = \sum_n p(n)[\, (S'(Y|n) - k \ln p(n)] = C$ where C is a device dependent constant; $E_n = E_m = E$

   $\Delta Q_{res} \geq 0 \quad \Delta W \geq 0$

….suggests that it is possible to design a computer to perform any combination of logical operations, with no exchange of heat with the environment and requires no work to be performed upon it. This must be as true for (all) logical operations

Logically deterministic, irreversible computations are able to avoid generating heat, in this model, by increasing the size of the physical states representing the logical states. This does *not* mean that the logical processing apparatus itself needs to be increasing in size. Although the size of the individual states has increased, the number of logical states has decreased (by the definition of a logically deterministic, irreversible computation!).

RLE + LE Cycle Abiabatic Computing

- RLE(p) (UFZ): input state 0 with certainty, but now the particle occupies the entire box
- Only one step, insert the barrier at x=p ⇒decrease in average state entropy by: p ln p + (1- p) ln (1- p) but this is compensated by the increase in the mixing entropy
- LE(p) (RTZ): input state 0 with probability p and 1 with probability 1-p
- Only one step, remove the barrier ⇒ increase in average state entropy by: p ln p + (1- p) ln (1- p) but this is compensated by the decrease in the mixing entropy

- M=M’; Y=Y’ and p(y|m) = p’(y|m) initial and final distributions inside logical subspace are the same ⇒ S(Y|m) = S’(Y|m)

- $\Delta S_{cond} = \sum_m$ [p’(m) – p(m)] (S(Y|m)

- If S(Y|m) is independent on m ⇒ΔS_{cond} = 0
  symmetric memories

- Do we need asymmetric memories?
  (talk of Dr. Chiuchiu’)

Conclusions

Can we perform logically irreversible computation in a thermodynamically reversible manner?

✓ Yes if we use Gibbs entropy $\Rightarrow \Delta S_{syst} = k\ \Delta H + \Delta S_{cond}$

Can we perform logically irreversible computation without dissipating heat in the environment?

✓ Yes, the decrease in the logical entropy must be compensated by the increase of the entropy inside logical subspace

Are these two questions two different questions?

✓ Thermodynamic reversibility $\not\Rightarrow$ no dissipation of heat

Thank you!